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A dynamic optimization method for determining cutoff grades in underground mines

Introduction

Cutoff grade is one of the most important parameters in mining because of its influence on the overall economic gains of a mining operation. Choosing the best cutoff grades that maximize the economic outcome has been a major topic of research since the 1960's, and many researchers have made contributions in devising various methods and algorithms.

Lane's work has been regarded as the landmark in cutoff grade optimization. His model takes the maximization of present value as the objective function and is able to consider the capacity constraints of mining, concentrating and refining stages as well as the capacity balancing of the three stages (Lane 1965). Hall et al. (1969) introduced the concept of "dual cutoff grades", one for determining whether to mine or leave and the other for determining whether to treat the mined out material as ore or waste. Dagdelen (1986) established the relationship between the dual cutoffs and the Lagrangian multipliers of reserve parameterization in his production scheduling model.

Cutoff grade also affects production planning through its influence on the ore quantity. Therefore, some researchers take cutoff grade as an internal decision variable in their production scheduling schemes which are either a Linear Programming or Dynamic Programming (Johnson 1968; Dowd 1976; Elbrond et al. 1982; Gershon 1982). These methods relate cutoff grade to the optimization of production capacity but not to the mining sequence.

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Much effort has been devoted to the cutoff grade determination in 1980's and 1990's. Many researchers developed methods and algorithms with different focuses and merits. It is not the intention of this paper to give a comprehensive review of the extensive literature in this field. After the turn of the century, however, not many publications have been seen in the literature on this topic. Works by Shinkuma & Nishiyama (2000), Ataei & Osanloo (2003), and Cairns & Shinkuma (2003) are examples of recent work on cutoff grade selection.

After many years of research, the cutoff grade determination is still by and large an unsolved problem. Two aspects must be considered in solving this problem: One is the dynamic nature of cutoff grade with respect to time, and the other is the dynamic nature of cutoff grade with respect to space. That is, the optimum cutoff grade varies with both time and the place of mining in the deposit and the two aspects are interrelated. The first aspect has been considered in Lane's work and some others, resulting in different cutoff grades in different mining periods.

However, the variation of mineral grades with space (i.e. zones mined in different periods) has not been fully considered. Because in almost all deposits different zones have different statistical grade distributions and average grades, ignoring such variation will result in unrealistic cutoff grades. For example, using Lane's method, the resulting cutoff grade decreases with time. In cases where the mineral grade is low in the upper part of the deposit and high in the lower part, using high cutoff grade in the early mining stage is obviously not appropriate and may not even be practical in some cases.

This paper proposed a Dynamic-Programming based method for optimizing cutoff grade in underground mines that can simultaneously consider the time and space dynamic nature of the cutoff grade problem.

1. The dynamic programming formulation

In underground mines, mining progresses level by level and stope by stope (or section by section) at a given level. In almost all cases, mineral grade varies with location, each zone having its local grade distribution whose mean and variance are different to a large or small extent from other zones. When the variation in local grade distribution is relatively high, using the overall grade distribution to determine the best cutoff grade in different mining periods (e.g. years) will be unrealistic and the results will not be optimum. In such cases, the cutoff grade selection for a particular mining period, a year, for example, must be related to the local grade distribution of the zone to be mined in that period. Furthermore, the cutoff grade for each mining period/zone should not be optimized as an isolated case independent of other periods/zones, because the cutoff grade in one period affects the cutoff decisions in later periods. Therefore, the problem must be solved in a dynamic fashion taking account of the local grade distribution in the decision-making process. This nature of the problem fits itself to a Dynamic Programming scheme.

1.1. Stages and states

The ultimate objective of cutoff grade optimization is to determine the best cutoff grade that should be used in mining each section (or zone). So each section is a decision-making unit and is taken as the stage in Dynamic Programming. Such units are referred to as "decision units" hereafter.

The portion of the deposit that has been explored and will be mined in the future years is divided into decision units, that is, stages. The delineation of the units, the sizes and locations of the units, is based on the mining method, the grade variation characteristics, and the sample data availability. If the grade variation is high and the sample spacing is small, a decision unit can be as small as a stope, or a small section of a mining level when the mining is not carried out in clearly defined stopes as in sublevel caving. If the grade is relatively stable with respect to location, a decision unit can be relatively big and the units are delineated in such a way so that the statistical grade distribution of each unit is clearly different from those of its neighboring units. When samples are sparse, a small unit will not contain enough samples to get a statistical grade distribution. In such cases, one way is to have big units and a better way is to have relatively small units and using the statistical grade distribution of a bigger section as the statistical grade distribution of each unit within the section. The important thing is that each unit has a grade distribution from which the grade-tonnage relationship can be calculated. The number of stages in the Dynamic Programming scheme is the same as the number of decision units.

A state of a given stage is defined as the cumulative quantity of ore mined from the beginning (the first decision unit) to the end of this stage when this state is reached. A stage has a number of states, each of which corresponding to a cutoff grade. Taking a copper mine as an example, suppose that the lowest and highest cutoff grades considered are 0.5% and 1.2%, respectively. From the grade-tonnage relationship of the first optimization unit, the ore quantities corresponding to cutoff grades of 0.5% and 1.2% can be obtained, say they are 2,000,000 t and 1,000,000 t, respectively. Using a step size of 100,000 t, there will be 11 states at the first stage. Supposing that, from the grade-tonnage relationship of the second decision unit, the ore quantities corresponding to cutoff grades of 0.5% and 1.2% are 800,000 t and 500,000 t, respectively, the cumulative ore quantity of the first two stages will range from 1,500,000 t to 2,800,000 t. With a step size of 100,000 t, there will be 14 states at the second stage. In this fashion, the states of all the stages can be determined.

1.2. The dynamic programming model

Once the stages and states are setup, they can then be put into a Dynamic Programming network as shown in Figure 1, where the horizontal axis represents the stage and the vertical axis represents the state. The sequence of the stages are same as the sequence of mining the corresponding decision units in the long-term mining schedule, i.e. the first stage corresponds to the unit that will be mined first and the second stage corresponds to the unit mined after the first, and so on. The states of each stage are arranged from the lowest cumulative ore quantity to the highest as represented by the circles in the Figure. Each arrow represents a possible transition. For the purpose of clarity, not all transitions are drawn in Figure 1.

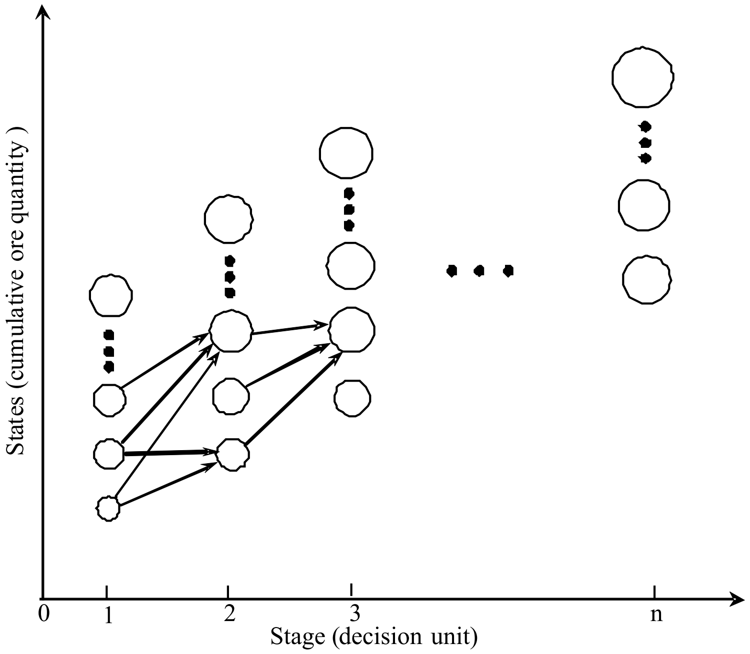

Fig. 1. Dynamic programming network for cutoff grade optimization

Rys. 1. Sieć programowania dynamicznego dla optymalizacji klasy odcięcia

For the convenience of description, the following symbols are defined:

n = the total number of stages,

i = stage sequence number, $i = 1, 2, \ldots n$,

$S_{i,j}$ = the jth state of stage i,

$Q_{i,j}$ = the cumulative ore quantity associated with state $S_{i,j}$, $Q_{i,j}$ are predetermined as described above when the states are setup,

$T_{i,j}$ = the total time length required to reach state $S_{i,j}$ from the beginning , following the best route,

$NPV_{i,j}$ = the total net present value achieved when state $S_{i,j}$ is reached following the best route,

$g_{i,j}$ = the best cutoff grade associated with state $S_{i,j}$,

d = discount rate,

m_i = ore production rate as planned for stage i (when mining decision unit i) which could be a constant,

C_m = unit cost of mining,

C_p = unit cost of ore processing,

R = the overall metal recovery rate of mining and ore processing,
G_p = grade of ore concentration,
p = price of ore concentration, which could be different in different stages.

For stage 1 (i = 1), the cutoff grade, $g_{1,j}$, associated with each state, $S_{1,j}$, is determined from the grade-tonnage relationship of decision unit 1, that is, the cutoff grade corresponding to ore quantity $Q_{1,j}$. Suppose that the final product is concentrated ore. The profit, $P_{1,j}$, associated with each state of stage 1 can be calculated by

$$P_{1,j} = \frac{Q_{1,j} G_{1,j} R}{G_p} p - Q_{1,j} (C_m + C_p) \tag{1}$$

Where $G_{1,j}$ = the average grade of $Q_{1,j}$, $G_{1,j}$ can be calculated from the statistical grade distribution of decision unit 1.

The profit calculation could be more comprehensive to account for more detailed cost items. The time length, $T_{1,j}$, required to reach state $S_{1,j}$ of the first stage (i.e. the time required to mine $Q_{1,j}$) is

$$T_{1,j} = \frac{Q_{1,j}}{m_1} \tag{2}$$

Then, the NPV associated with each state of the first stage is

$$NPV_{1,j} = \frac{P_{1,j}}{(1+d)^{T_{1,j}}} \tag{3}$$

This is a simple calculation for illustration purposes. $T_{1,j}$ may not be integer number of years, and a more realistic way is to distribute the profit evenly on a yearly basis.

After the above calculation for all the states of the first stage, the process goes to the second stage. Since the state variable is defined as the cumulative quantity of ore mined when a state is reached, each state of stage 2 can only be reached from those states of stage 1 whose ore quantities are smaller than the ore quantity associated with the state of stage 2. Taking the 3rd state of stage 2, $S_{2,3}$, as an example, it can only be reached from the 3 states of stage 1, as shown by the 3 arrows in Figure 1.

Generally, when state j of stage i, $S_{i,j}$, is reached from state k of the previous stage, $S_{i\text{-}1,k}$, the ore quantity mined in stage i for such a transition is denoted by $q_{i,j}(i\text{-}1,k)$, the average grade of this ore is denoted by $G_{i,j}(i\text{-}1, k)$, the time length required to mine $q_{i,j}(i\text{-}1,k)$ quantity of ore is denoted by $t_{i,j}(i\text{-}1,k)$, and the associated profit realized for such a transition is $P_{i,j}(i\text{-}1,k)$.

Now, consider the case of reaching state $S_{2,3}$ in stage 2 from state $S_{1,1}$ in stage 1, the ore quantity mined in stage 2 (i.e. mined in decision unit 2) for such a transition is $q_{2,3}(1,1)$ and calculated by

$$q_{2,3}(1,1) = Q_{2,3} - Q_{1,1} \tag{4}$$

From the grade-tonnage relationship and the statistical grade distribution of decision unit 2, the cutoff grade corresponding to this ore quantity, $q_{2,3}(1, 1)$, and the average ore grade, $G_{2,3}(1, 1)$, can be obtained. Then, the profit of mining $q_{2,3}(1, 1)$ in stage 2 is $P_{2,3}(1, 1)$, which can be computed by applying Equation 1 with $q_{2,3}(1, 1)$ replacing $Q_{1,j}$ and $G_{2,3}(1, 1)$ replacing $G_{1,j}$. The time length of mining $q_{2,3}(1, 1)$ is $t_{2,3}(1, 1) = q_{2,3}(1, 1)/ m_2$. Then the total NPV at state $S_{2,3}$ when reached from $S_{1,1}$ is

$$NPV_{2,3}(1,1) = NPV_{1,1} + \frac{P_{2,3}(1,1)}{(1+d)^{T_{1,1}+t_{2,3}(1,1)}} \tag{5}$$

It is not hard to understand that, when state $S_{2,3}$ is reached from a different state of stage 1, the ore quantity mined in stage 2 will be different and the cutoff grade corresponding to the new ore quantity will be different from the previous transition. Consequently, the total net present value at state $S_{2,3}$ will also be different. Therefore, for this given state $S_{2,3}$, each transition has a cutoff grade and NPV. The one with the highest NPV is the best transition and the associated cutoff grade is the best cutoff grade, $g_{2,3}$, for state $S_{2,3}$. Therefore, we have

$$NPV_{2,3} = \max_{k \in K_{2,3}} \{NPV_{2,3}(1,k)\} = \max_{k \in K_{2,3}} \left\{ NPV_{1,k} + \frac{P_{2,3}(1,k)}{(1+d)^{T_{1,k}+t_{2,3}(1,k)}} \right\} \tag{6}$$

Where $K_{2,3}$ is the number of states in stage 1 from which state $S_{2,3}$ in stage 2 can be reached.

In this fashion, the best transitions and the associated best cutoff grades and NPVs can be obtained for all the states in stage 2.

After the above process for stage 2 is repeated for all the remaining stages, the best transitions and the associated best cutoff grades and NPVs are calculated for all the states of all stages in Figure 1. Then starting from the state of the last stage that has the highest NPV of all states of the same stage, tracing the best transitions back to the first stage, the optimum route can be found, which is called the optimum policy in Dynamic Programming. This optimum policy indicates the best cutoff grade to be used, the corresponding ore quantity and economic gains in mining each of the decision unit.

Clearly, it is a formulation of the forward Dynamic Programming algorithm. In a general form, when state $S_{i,j}$ of a stage is reached from state $S_{i\text{-}1,k}$ of the previous stage, the transition function is given by Equation 7 below, which is the ore quantity mined in decision unit *i* when such a transition is made.

$$q_{i,j}(i-1,k) = Q_{i,j} - Q_{i-1,k} \tag{7}$$

This equation establishes the link between state *j* of stage *i* and state *k* of stage *i*-1.

A simplified calculation for the profit, $P_{i,j}(i\text{-}1,k)$, made when state $S_{i,j}$ of a stage is reached from state $S_{i\text{-}1,k}$ of the previous stage, is given by

$$P_{i,j}(i-1,k) = \frac{q_{i,j}(i-1,k)\,G_{i,j}(i-1,k)R}{G_p}\,p - q_{i,j}(i-1,k)(C_m + C_p) \tag{8}$$

The equation for calculating the time, $t_{i,j}(i\text{-}1,k)$, required to mine $q_{i,j}(i\text{-}1,k)$ quantity of ore in decision unit *i* is

$$t_{i,j}(i-1,k) = \frac{q_{i,j}(i-1,k)}{m_i} \tag{9}$$

The total time to arrive at state $S_{i,j}$ when it is reached from state $S_{i\text{-}1,k}$ is denoted by $T_{i,j}(i\text{-}1,k)$ and calculated as

$$T_{i,j}(i-1,k) = T_{i-1,k} + t_{i,j}(i-1,k) \tag{10}$$

A simplified calculation for the total NPV realized up to state $S_{i,j}$ after mining ore amount, $q_{i,j}(i\text{-}1,k)$, in decision unit *i*, following such a transition is

$$NPV_{i,j}(i-1,k) = NPV_{i-1,k} + \frac{P_{i,j}(i-1,k)}{(1+d)^{T_{i,j}(i-1,k)}} \tag{11}$$

The recursive function can then be written as follows

$$NPV_{i,j} = \max_{k\in K_{i,j}} \{NPV_{i,j}(i-1,k)\} = \max_{k\in K_{i,j}} \left\{ NPV_{i-1,k} + \frac{P_{i,j}(i-1,k)}{(1+d)^{T_{i,j}(i-1,k)}} \right\} \tag{12}$$

Where $K_{i,j}$ is the number of states in stage *i*-1 from which state $S_{i,j}$ in stage *i* can be reached.

1.3. The algorithm

Based on the above model, the algorithm for optimizing cutoff grade in underground mines is as follows:

Step 1. Divide the portion of the deposit, for which the mining method has been selected and mining schedule has been done, into proper decision units as described in Section 2.1 above. The number of stages, n, is equal to the number of decision units. Also, prepare the necessary input data required in calculating profit and NPV, such as costs, prices, recovery rates, production rates, and discount rate, which can be constants or vary with time.

Step 2. From the sampling data falling in each decision unit, prepare a table of ore quantity and average ore grade as a function of cutoff grade. This table is hereafter referred to as the "tonnage-grade relationship". Lower and upper bounds, g_L and g_U, can be used as constraints on cutoff grade. For a copper mine, for example, g_L and g_U could be 0.5% and 1.5%, respectively.

Step 3. As described in Section 2.1, determine the number of states of each stage and the cumulative ore quantity, $Q_{i,j}$, associated with each state.

Step 4. Set $i = 1$, i.e. consider the first stage. For each state of this stage, get the cutoff grade and the average ore grade, from the tonnage-grade relationship, corresponding to the ore quantity of the state. Then, calculate the profit, the mining time, and the NPV using Equations 1–3.

Step 5. Increment the stage index i by 1. Starting from the lowest state ($j = 1$), do the evaluation in the following Step.

Step 6. For a state $S_{i,j}$, consider a transition from a state $S_{i-1,k}$ of the previous stage. Do the following:

a) Calculate the ore quantity of such a transition using Equation 7.

b) From the tonnage-grade relationship of decision unit i, get the cutoff grade and the average ore grade corresponding to the ore quantity. If the cutoff grade falls out of the preset lower and upper bounds, this transition is considered infeasible and go to the next transition (next k).

c) Calculate the profit, time, and total NPV corresponding to this transition using Equations 8–11.

Repeating a–c for all the possible transitions ($k = 1, 2, \ldots, K_{i,j}$). Then select the best transition which has the highest total NPV using Equation 12. Thus, the best cutoff grade and other parameters are obtained for state $S_{i,j}$.

Step 7. Increment the state index j by 1 and repeat Step 6, until the last state ($j = n_i$, n_i being the number of states in stage i) of stage i is evaluated.

Step 8. Repeat Steps 5&6 above, until the last stage ($i = n$) is completed.

Step 9. Select the state with the highest total NPV in the last stage. Starting from this state, trace backward the best transitions stage by stage, until the first stage is reached. The resulting route is the optimum policy. This policy indicates the optimum cutoff grade to be used in each of the n decision units.

Conclusions

A Dynamic-Programming based model has been developed and the algorithm given for the optimization of cutoff grade in underground mines. The merit of the model lies in that the local grade variation in a deposit is fully accounted for. The algorithm is robust in that relevant constraints can easily be incorporated and comprehensive economic evaluation can be developed and used in the decision-making process. The model and the algorithm can also be adapted to optimizing cutoff grade in open-pit mines with small changes. The only difference when applied to open pit mines is the delineation of the decision units. The model has been successfully applied to Chambishi copper mine in Zambia.

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METODA OPTYMALIZACJI DYNAMICZNEJ DLA OKREŚLENIA KLASY ODCIĘCIA W KOPALNIACH PODZIEMNYCH

Słowa kluczowe

Klasy odcięcia, metoda optymalizacji dynamicznej, Programowanie Liniowe, kopalnie podziemne [red.]

Streszczenie

Optymalizacja klasy odcięcia jest starym tematem, który był analizowany przez wielu badaczy. Jednak istniejące metody nie mogą znaleźć realnego i optymalnego rozwiązania, ponieważ opierają się na rozkładzie gatunku całego złoża, natomiast rozkłady gatunku różnych sekcji złoża mogą znacznie różnić się od rozkładów całego złoża. Zaproponowano nową metodę będącą środkiem zaradczym dla tej niedoskonałości. Metoda ta rozwiązuje problem w trzech krokach: 1) podzielenie całego złoża na „jednostki decyzyjne" oparte na metodzie wydobycia i długoterminowym programie wydobycia, 2) obliczenie rozkładu gatunku każdej jednostki na podstawie próbek z tej jednostki i 3) uznanie każdej jednostki z jej gatunkami za etap w programie Dynamicznego Programowania i rozwiązanie problemu poprzez zastosowanie algorytmu Programowania Dynamicznego. Ta nowa metoda została pomyślnie zastosowana w kopalni miedzi Chambishi.

A DYNAMIC OPTIMIZATION METHOD FOR DETERMINING CUTOFF GRADES IN UNDERGROUND MINES

Key words

Cutoff grade, dynamic optimization method, Linear Programming, underground mines [red.]

Abstract

Cutoff grade optimization is an old topic that has been studied by many researchers. However, existing methods cannot find the real optimum solution because they are based on the grade distribution of the entire deposit, while the grade distributions of different sections of the deposit may be much different from that of the entire deposit. A new method is proposed to overcome this shortcoming. This method solves the problem in three steps: 1) Divides the entire deposit into "decision units" based on the mining method and long term mining schedule, 2) the grade distribution of each unit is computed based on the samples falling in the unit, and 3) each unit with its grade distribution is considered as a stage in a Dynamic Programming scheme and the problem is solved by applying the Dynamic Programming algorithm. The new method hes been successfully applied to the Chambishi copper mine.